Solving Absolute Value Equations

Solving an Absolute Value Equation

Step 1 Isolate the absolute value. Write the equation in the form $|x| = a$, where a is a real number.

Step 2 If $a < 0$, there is no solution.

Step 3 If $a \geq 0$, rewrite the absolute value equation as $x = a$ or $x = -a$.

Step 4 Solve the individual equations from step 3.

Step 5 Check the answers in the original absolute value equation.

1. Solve. $|3x-2| = 16$

Step 1: $|3x-2| = 16$

Step 2: $16 > 0$, THERE IS A SOLUTION.

Step 3: $3x-2 = 16$ OR $3x-2 = -16$

Step 4:

$3x-2 = 16$ OR $3x-2 = -16$

$3x = 18$ $3x = -14$

$x = 6$ OR $x = -14/3$

Step 5:

$|3(6)-2| = 16$

$|18-2| = 16$ ✓

$|3(-14/3)-2| = 16$

$|-14-2| = 16$ ✓

2. Solve. $|2x-1|+2=5$

$|2x-1| = 5-2 \Rightarrow |2x-1| = 3$

$2x-1 = 3$ OR $2x-1 = -3$

$2x = 4$ $\quad$ $2x = -2$

$x = 2$ OR $x = -1$

CHECK:

$|2(2)-1|=3$ $\quad$ $|2(-1)-1|=3$

$|4-1|=3$ ✓ $\quad$ $|-2-1|=3$ ✓

3. Solve. $\left|3+\frac{1}{4}x\right|=4$ $\Rightarrow$ $3+\frac{1}{4}x = 4$ OR $3+\frac{1}{4}x = -4$

$4\left(\frac{1}{4}x\right) = 4(1)$ $\quad$ $4\left(\frac{1}{4}x\right) = 4(-7)$

$x = 4$ OR $x = -28$

CHECK:

$\left|3+\frac{1}{4}(4)\right| = 4$ OR $\left|3+\frac{1}{4}(-28)\right| = 4$

$|3+1| = 4$ ✓ OR $|3-7| = 4$ ✓

4. Solve. $\frac{1}{2}\left|x+\frac{1}{4}\right|=3$ $\Rightarrow$ $2\left(\frac{1}{2}\left|x+\frac{1}{4}\right|\right) = 2(3)$

$\left|x+\frac{1}{4}\right| = 6 \Rightarrow x+\frac{1}{4} = 6$ OR $x+\frac{1}{4} = -6$

$4(x)+4\left(\frac{1}{4}\right) = 4(6)$ OR $4(x)+4\left(\frac{1}{4}\right) = 4(-6)$

$4x+1 = 24$ OR $4x+1 = -24$

$4x = 23$ $\quad$ $4x = -25$

$x = 23/4$ OR $x = -25/4$

CHECK:

$\frac{1}{2}\left|\frac{23}{4}+\frac{1}{4}\right|=3$ $\quad$ $\frac{1}{2}\left|\frac{-25}{4}+\frac{1}{4}\right|=3$

$\frac{1}{2}|6| = 3$ ✓ $\quad$ $\frac{1}{2}|6| = 3$ ✓

5. Solve. $-3\left|x-\frac{1}{2}\right|-7=-13$

$-3\left|x-\frac{1}{2}\right| = -13+7$

$\left|x-\frac{1}{2}\right| = \frac{-6}{-3}$

$\left|x-\frac{1}{2}\right| = 2$

$x-\frac{1}{2}=2$ OR $x-\frac{1}{2}=-2$

$2(x)-2\left(\frac{1}{2}\right)=2(2)$ OR $2(x)-2\left(\frac{1}{2}\right)=2(-2)$

$2x-1=4$ OR $2x-1=-4$

$2x=5$ OR $2x=-3$

$x=5/2$ OR $x=-3/2$

CHECK: $-3\left|\frac{5}{2}-\frac{1}{2}\right|-7=-13 \Rightarrow -3|2|-7=-13$ ✓ { $-3\left|-\frac{3}{2}-\frac{1}{2}\right|-7=-13 \Rightarrow -3|-2|-7=-13$ ✓

6. $|8x+9|+4=2$

$|8x+9| = -2$

NO SOLUTION.

Solving Equations Containing Two Absolute Values

$|x|=|y| \rightarrow x=y$ or $x=-y$

7. Solve. $|2x-5|=|3x+7|$

$2x-5=3x+7$ OR $2x-5=-(3x+7)$

$2x-3x=7+5$ $2x-5=-3x-7$

$-x=12$ $2x+3x=-7+5$

$x=-12$ $5x=-2$

OR $x=-2/5$

CHECK:

$|2(-12)-5| = |3(-12)+7|$

$|-29| = |-29|$ ✓

$\left|2\left(-\frac{2}{5}\right)-5\right| = \left|3\left(-\frac{2}{5}\right)+7\right|$

$\left|-\frac{4}{5}-\frac{5}{1}\right| = \left|-\frac{6}{5}+\frac{7}{1}\right|$

$\left|\frac{-4-25}{5}\right| = \left|\frac{-6+35}{5}\right|$ ✓

8. Solve. $\left|\frac{1}{2}x+2\right| = \left|\frac{3}{4}x-2\right|$

$\frac{1}{2}x+2 = \frac{3}{4}x-2$ OR $\frac{1}{2}x+2 = -\left(\frac{3}{4}x-2\right)$

$4\left(\frac{1}{2}x\right)+4(2) = 4\left(\frac{3}{4}x\right)-4(2)$ OR $4\left(\frac{1}{2}x+2\right) = 4\left(-\frac{3}{4}x\right)+4(2)$

$2x+8 = 3x-8$ OR $2x+8 = -3x+8$

$2x-3x = -8-8$ $\quad$ $2x+3x = 8-8$

$-x = -16$ $\quad$ $5x = 0$

$x = 16$ $\quad$ OR $\quad$ $x = 0$

CHECK:

$\left|\frac{1}{2}(16)+2\right| = \left|\frac{3}{4}(16)-2\right|$

$|8+2| = |12-2|$ ✓

$\left|\frac{1}{2}(0)+2\right| = \left|\frac{3}{4}(0)-2\right|$

$|2| = |-2|$ ✓

9. Solve. $\left|\frac{3x-2}{4}+1\right|=\left|\frac{x+1}{5}\right|$

$\frac{3x-2}{4}+1=\frac{x+1}{5}$ OR $\frac{3x-2}{4}+1=-\left(\frac{x+1}{5}\right)$

$20\left(\frac{3x-2}{4}\right)+20(1)=20\left(\frac{x+1}{5}\right)$ OR $20\left(\frac{3x-2}{4}\right)+20(1)=-20\left(\frac{x+1}{5}\right)$

$5(3x-2)+20=4(x+1)$ OR $5(3x-2)+20=-4(x+1)$

$15x-10+20=4x+4$ $15x-10+20=-4x-4$

$15x+10=4x+4$ $15x+10=-4x-4$

$15x-4x=-10+4$ $15x+4x=-4-10$

$11x=-6$ $19x=-14$

$\boxed{x=-6/11}$ $\boxed{x=-14/19}$

CHECK:

$\left|\frac{3\left(\frac{-6}{11}\right)-2}{4}+1\right|=\left|\frac{\left(\frac{-6}{11}\right)+1}{5}\right|$

$\left|\frac{1}{11}\right|=\left|\frac{-6+11}{55}\right|$

$\left|\frac{1}{11}\right|=\left|\frac{1}{11}\right|$ ✓

$\left|\frac{3\left(\frac{-14}{19}\right)-2}{4}+1\right|=\left|\frac{\left(\frac{-14}{19}\right)+1}{5}\right|$

$\left|\frac{-42-38}{76}+\frac{76}{76}\right|=\left|\frac{-14+19}{95}\right|$

$\left|-\frac{1}{19}\right|=\left|\frac{1}{19}\right|$ ✓

10. $|7x-2|=-\left|x+\frac{1}{2}\right|$

↓

ABSOLUTE VALUE CANNOT EQUAL A NEGATIVE QUANTITY.
THEREFORE, THERE IS NO SOLUTION.

Solving Absolute Value Inequalities by Definition

Solving Absolute Value Equations and Inequalities

Let a be a real number such that $a > 0$. Then

Equation/ Inequality	Solution (Equivalent Form)	Graph
$\|x\| = a$	$x = -a$ or $x = a$	
$\|x\| > a$	$x < -a$ or $x > a$	
$\|x\| < a$	$-a < x < a$	

Example:

Solve. $\left|3+\frac{1}{4}x\right| \le 4$ →

$-4 \le 3+\frac{1}{4}x \le 4$ MULT. BY 4.

$-16 \le 12+x \le 16$

$-12 \quad -12 \quad -12$ SUBTRACT 12

$-28 \le x \le 4$

ANS. $[-28, 4]$

1. Solve. $-3\left|x-\frac{1}{2}\right|-7 > -13$ → $-3\left|x-\frac{1}{2}\right| > -13+7$

→ $-3\left|x-\frac{1}{2}\right| > -6$ → $\left|x-\frac{1}{2}\right| < \frac{-6}{-3}$

$\left|x-\frac{1}{2}\right| < 2$

$-2 < x-\frac{1}{2} < 2$ (ADD $\frac{1}{2}$) → $-2+\frac{1}{2} < x-\frac{1}{2}+\frac{1}{2} < 2+\frac{1}{2}$

ANS. $(-3/2, 5/2)$ $-\frac{3}{2} < x < 5/2$

2. Solve. $|5x+7| \le 2$ →

$-2 \le 5x+7 \le 2$

$-7 \quad -7 \quad -7$

$-9 \le 5x \le -5$

$\frac{-9}{5} \le \frac{5x}{5} \le \frac{-5}{5}$

$-9/5 \le x \le -1$

Ans. $[-9/5, -1]$

Solving Absolute Value Inequalities by the Test Point Method

Solving an Absolute Value Inequality

Step 1 Find the boundary points of the inequality. (Solve the related equation.)

Step 2 Plot the boundary points on the number line. This divides the number line into intervals.

Step 3 Select a test point from each interval and substitute it into the original inequality.

- If a test point makes the original inequality true, then **that interval is part of the solution set**.

Step 4 Test the boundary points in the original inequality.

- If the original inequality is strict (< or >) , do not include the boundary points in the solution set.
- If the original inequality is defined using ≤ or ≥ , then include the boundary points that are well defined within the inequality.

Note: Any boundary point that makes an expression within the inequality undefined must *always* be excluded from the solution set.

3. Solve. $|3x-2| < 16$

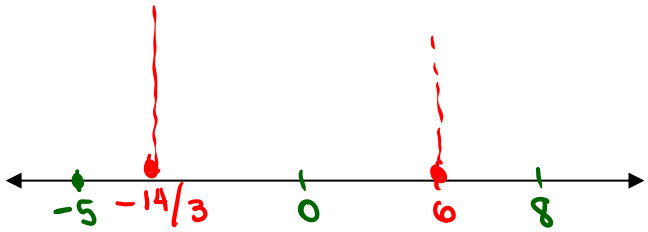
Step 1:

$3x-2 = 16$ OR $3x-2 = -16$

$3x = 18$ $3x = -14$

$x = 6$ $x = -14/3$ BOUNDRY POINTS

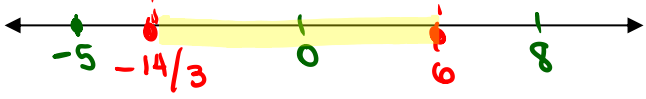
Step 2:



Step 3: TEST POINTS: $-5, 0, 8$.

Step 4: $|3(-5)-2|<16$ FALSE; $|3(0)-2|<16$ TRUE; $|3(8)-2|<16$ FALSE

Step 5:



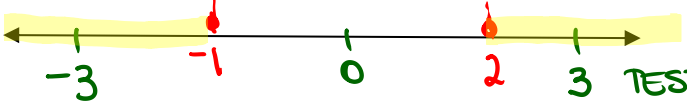
$(-14/3, 6)$

4. Solve. $|2x-1|+2 \geq 5 \rightarrow |2x-1| \geq 3 \rightarrow 2x-1=3$ OR $2x-1=-3$

$2x=4 \quad 2x=-2$

BOUNDRY POINTS $\rightarrow x=2$ OR $x=-1$

$|2(-3)-1| \geq 3$ TRUE; $|2(0)-1| \geq 3$ FALSE; $|2(3)-1| \geq 3$ TRUE

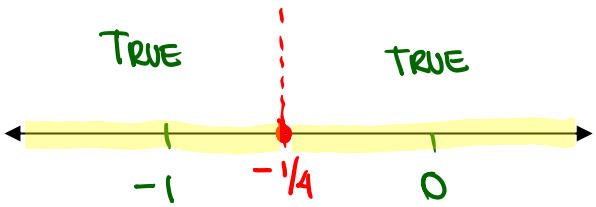


ANS $(-\infty, -1] \cup [2, \infty)$

5. Solve. $\frac{1}{2}\left|x+\frac{1}{4}\right|>0 \rightarrow \left|x+\frac{1}{4}\right|>0 \rightarrow x+\frac{1}{4}=0$

$x=-1/4$ BOUNDRY POINT.

TRUE; TRUE



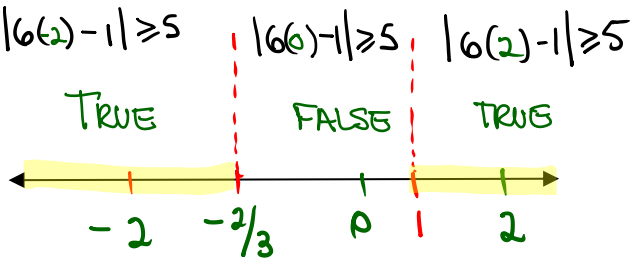
ANS. $(-\infty, -1/4) \cup (-1/4, \infty)$

(ALL REAL NUMBERS, EXCEPT $x=-1/4$)

6. Solve. $2|6x-1| \geq 10 \rightarrow |6x-1| \geq 5 \rightarrow 6x-1=5$ OR $6x-1=-5$

$6x=6 \quad 6x=-4$

$x=1$



$x = -4/6$

$x = -2/3$

Ans. $(-\infty, -2/3] \cup [1, \infty)$

Translating to an Absolute Value Expression

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7. When a number is subtracted from –7, the absolute value of the difference is more than 3. Use interval notation to express the set of all numbers that satisfy this condition.

$|-7 - x| > 3 \Rightarrow -7 - x > 3$ OR $-7 - x < -3$

$-x > 10$ $\qquad -x < 4$

$x < -10$ OR $x > -4$

$(-\infty, 10) \cup (-4, \infty)$

8. A landscaping company sells 40-pound bags of top soil. The actual weight *x* of a bag, however, may differ from the advertised weight by as much as 0.75 pound. Write an inequality involving absolute value that expresses the relationship between the actual weight *x* of a bag and 40 pounds. Solve the inequality and express the answer in interval form.

$|40 - x| \leq 0.75$

$-0.75 \leq 40 - x \leq 0.75$

$-40.75 \leq -x \leq -39.25$ (MULT. BY -1.)

$40.75 \geq x \geq 39.25 \Rightarrow 39.25 \leq x \leq 40.75$

$[39.25, 40.75]$